The aim of this monograph is to demonstrate the application of algebra and algebraic number theory to the study of combinatorics problems. Three problems which will be discussed in this monograph are Group-Invariant Butson Hadamard Matrices, Unique Differences in Symmetric Subsets of $\mathbb{F}_p$ and Upper Bounds for Cyclotomic Numbers. The main contribution of this thesis is the construction of new classes of the mentioned objects and better necessary conditions (than the known ones) for their existence. In some cases, our necessary conditions are also sufficient conditions or asymptotically best conditions.