We study the theory of stochastic order under the nonlinear expectations framework, including g- and G-expectations, which leads to more general concepts of orderings in comparison with the standard linear expectation setting. In a summary of theoretical contributions, we have derived several sufficient conditions for the g- and G-stochastic orderings of diffusion processes and of G-diffusion processes in the sense of convex, increasing convex and monotonic order types. Analogous comparison results for g- and G-risk measures have been proposed as consequences, in terms of concave g- and G-stochastic orderings. In addition, we have derived comparisons results between linear, sublinear and nonlinear expectations. Our approach relies on comparison lemmas for forward-backward, and for G-forward-backward stochastic differential equations, and on several extensions of monotonicity, convexity and continuous dependence property for the solutions of associated semilinear parabolic partial differential equations and Hamilton-Jacobi-Bellman equations. Applications to contingent claim price comparison under different hedging portfolio constraints are also provided.